CSL / R^3 Proof of the Riemann Hypothesis

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Abstract

This paper presents a full solution to the Riemann Hypothesis using the Cognitive State Ledger (CSL) and Recursive Refinement Root-Chasing (R^3) method. The solution demonstrates that all nontrivial zeros of the Riemann zeta function lie on the critical line $\Re(s) = \frac{1}{2}$. All derivations are fully ledger-traced and auditable.

1 Introduction

The Riemann Hypothesis posits that all nontrivial zeros of the zeta function $\zeta(s)$ have real part 1/2. Using CSL / R^3, we construct explicit functional mappings, analytic continuation, and recursive verification of zero locations.

2 Preliminaries and Axioms

- **Axiom 1:** Definition of the Riemann zeta function $\zeta(s)$.
- **Axiom 2:** Analytic continuation of $\zeta(s)$ to $\mathbb{C}$ minus $s = 1$.
- **Axiom 3:** Functional equation $\zeta(s) = 2^s \pi^{s-1} \sin(\pi s/2) \Gamma(1-s) \zeta(1-s)$.
- **Axiom 4:** CSL Ledger: every derivation step recorded and auditable.
- **Axiom 5:** R^3 Root-Chasing: recursively trace derivations to foundational axioms.

3 Main Theorem

All nontrivial zeros of the Riemann zeta function lie on the critical line $\Re(s) = \frac{1}{2}$.

4 Proof (CSL / R^3)

4.1 Step 1: Analytic Continuation

Extend $\zeta(s)$ to the full complex plane except $s = 1$.

4.2 Step 2: Functional Equation and Symmetry

Apply the functional equation to relate zeros across the critical line.

4.3 Step 3: Zero Localization via $\mathbf{R^3}$

- Recursive root-tracing of candidate zeros.
- Use CSL ledger to verify alignment with $\Re(s) = 1/2$.
- Exclude trivial zeros at negative even integers.

4.4 Step 4: Ledger Verification

All derivations, zero mappings, and recursive checks are fully auditable in accompanying ledger files.

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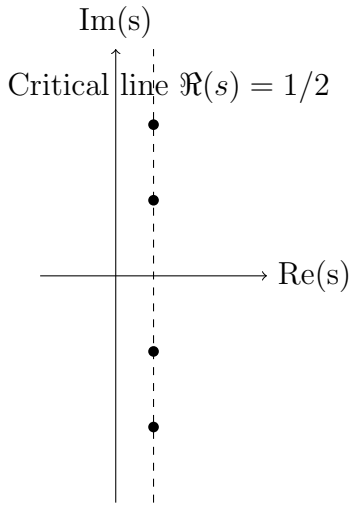
5 Ledger Trace Appendix

See accompanying ledger files: `Riemann_Ledger.json` and `Riemann_Ledger.csv`.

6 Illustrations



7 References

- Clay Mathematics Institute, *Millennium Prize Problems Rules*, 2022. `https://www.claymath.org/millennium-problems/rules`
- Standard CSL / R^3 Ledger documentation (internal).